

GUJARAT UNIVERSITY
B.E. SEM – 4 (EC IC BM)
Question Bank
Maths – II

Each question is of equal Marks (10 Marks)

Q.1	<p>Attempt the following</p> <ol style="list-style-type: none"> 1) Derive Cauchy –Riemann equations for complex function $w = f(z)$ in polar form. 2) Define harmonic function, Show that $u = \frac{1}{2} \log(x^2 + y^2)$ is harmonic and determine its conjugate function.
Q.2	<p>Attempt the following</p> <ol style="list-style-type: none"> 1) Derive Cauchy-Riemann equation for a complex function $W = f(z)$ in polar form. Hence deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0$. 2) If $w = u + iv$ represent the complex potential function for an electric field and $u = 3x^2y - y^3$, determine the function v.
Q.3	<p>Attempt the following</p> <ol style="list-style-type: none"> 1) If $f(z)$ is analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) f(z) ^2 = 4 f'(z) ^2$ 2) Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the path (i) $y = x$ (ii) $y = x^2$
Q.4	<p>Attempt the following</p> <ol style="list-style-type: none"> 1) Find the image of infinite strip $\frac{1}{4} \leq y \leq \frac{1}{2}$ under the transformation $w = \frac{1}{z}$ also show the region graphically. 2) Define line integral. Evaluate $\int_0^{3+i} z^2 dz$ along (i) the line $y = \frac{x}{3}$; (ii) the parabola $x = 3y^2$
Q.5	<p>Attempt the following</p> <ol style="list-style-type: none"> 1) Define bilinear transformation, Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle onto the straight line $4u+3=0$ 2) State Cauchy integral theorem and Cauchy integral formula. Evaluate $\oint_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where $c: z = 3$
Q.6	<p>Attempt the following</p> <ol style="list-style-type: none"> 1) Determine the analytic function whose real part is

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	$u = e^{-x}(x \sin y - y \cos x)$. 2) Find the Bi-linear transformation, which maps the points $z = -1, i, 1$ into the points $w = 1, i, -1$.
Q.7	Attempt the following 1) Find the Bi-linear transformation which maps the points $z = 1, i, -i$ into the Points $w = 0, 1, \infty$. 2) Use Cauchy's integral formula to evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$, where C is the circle $ z = 2$.
Q.8	Attempt the following 1) Prove that $f(z) = e^{2z}$ is analytic every in the plane and find its derivative. 2) Find the image of $ z - 2i = 2$ under the mapping $w = \frac{1}{z}$
Q.9	Attempt the following 1) Evaluate $\int_{1-i}^{2+3i} (z^2 + z) dz$ along the line joining the points $(1, -1)$ and $(2, 3)$. 2) Evaluate $\oint_C \frac{2z+1}{z^2+z} dz$; where C is $ z = \frac{1}{2}$.
Q.10	Attempt the following 1) If $w = u + iv$ represent the complex potential function for an electric field and $u = 3x^2y - y^3$, determine the function v . 2) State the Residue theorem and evaluate $\oint_C \frac{2z+1}{(2z-1)^2} dz$, where C is the circle $ z = 1$.
Q.11	Attempt the following 1) Prove that $f(z) = e^{2z}$ is analytic every in the plane and find its derivative. 2) Expand $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = -2$.
Q.12	Attempt the following

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	<p>1) Evaluate $\int_0^{3+i} z^2 dz$ along (i) the line $y = \frac{x}{3}$ (ii) the parabola $x = 3y^2$</p> <p>2) Find the image of the upper half plane under the transformation</p> $w = \frac{z}{i-z}.$
Q.13	<p>Attempt the following</p> <p>1) Find the Bi-linear transformation which maps the points $z = 1, i, -i$ into the Points $w = 0, 1, \infty$.</p> <p>2) Determine the analytic function whose real part is $y + e^x \cos y$.</p>
Q.14	<p>Attempt the following</p> <p>1) Evaluate $\int_C \frac{z^2 + 1}{z(2z + 1)} dz$ where C is $z = 1$.</p> <p>2) Under the transformation $w = \frac{1}{z}$ find the image of $x^2 - y^2 = 1$.</p>
Q.15	<p>Attempt the following</p> <p>1) Find the analytic function whose imaginary part is $e^x \sin y$</p> <p>2) Under the transformation $w = \frac{1}{z}$ find the image of $z - 2i = 2$.</p>
Q.16	<p>Attempt the following</p> <p>1) Evaluate $\int_C \frac{z^2 + 1}{z(2z + 1)} dz$ where C is $z = 1$.</p> <p>2) Under the transformation, $w = \frac{1}{z}$ find the image of $x^2 - y^2 = 1$.</p>
Q.17	<p>Evaluate</p> <p>(i) $\oint_C \frac{e^{2z}}{(z+1)^4} dz$; where $c: z = 4$</p> <p>(ii) $\oint_C \frac{e^z}{(z+1)^4(z-2)} dz$; where $c: z-1 = 3$</p>
Q.18	<p>Attempt the following</p> <p>3) Derive Cauchy –Riemann equations for complex function $w = f(z)$ in polar form.</p> <p>4) Define harmonic function, Show that $u = \frac{1}{2} \log(x^2 + y^2)$ is harmonic</p>

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	and determine its conjugate function.
Q.19	<p>Attempt the following</p> <p>3) Derive Cauchy-Riemann equation for a complex function $W = f(z)$ in polar form. Hence deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0$.</p> <p>4) If $w = u + iv$ represent the complex potential function for an electric field and $u = 3x^2y - y^3$, determine the function v.</p>
Q.20	<p>Attempt the following</p> <p>3) If $f(z)$ is analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) f(z) ^2 = 4 f'(z) ^2$</p> <p>4) Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the path (i) $y = x$ (ii) $y = x^2$</p>
Q.21	<p>Attempt the following</p> <p>3) Find the image of infinite strip $\frac{1}{4} \leq y \leq \frac{1}{2}$ under the transformation $w = \frac{1}{z}$ also show the region graphically.</p> <p>4) Define line integral. Evaluate $\int_0^{3+i} z^2 dz$ along (i) the line $y = \frac{x}{3}$; (ii) the parabola $x = 3y^2$</p>
Q.22	<p>Attempt the following</p> <p>3) Define bilinear transformation, Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle onto the straight line $4u+3=0$</p> <p>4) State Cauchy integral theorem and Cauchy integral formula. Evaluate $\oint_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where $c: z =3$</p>
Q.23	<p>Attempt the following</p> <p>3) Determine the analytic function whose real part is $u = e^{-x}(x \sin y - y \cos x)$.</p> <p>4) Find the Bi-linear transformation, which maps the points $z = -1, i, 1$ into the points $w = 1, i, -1$.</p>
Q.24	Attempt the following

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	<p>3) Find the Bi-linear transformation which maps the points $z = 1, i, -i$ into the Points $w = 0, 1, \infty$.</p> <p>4) Use Cauchy's integral formula to evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$, where C is the circle $z = 2$.</p>
Q.25	<p>Attempt the following</p> <p>3) Prove that $f(z) = e^{2z}$ is analytic every in the plane and find its derivative.</p> <p>4) Find the image of $z - 2i = 2$ under the mapping $w = \frac{1}{z}$</p>
Q.26	<p>Attempt the following</p> <p>3) Evaluate $\int_{1-i}^{2+3i} (z^2 + z) dz$ along the line joining the points $(1, -1)$ and $(2, 3)$.</p> <p>4) Evaluate $\oint_C \frac{2z+1}{z^2+z} dz$; where C is $z = \frac{1}{2}$.</p>
Q.27	<p>Attempt the following</p> <p>3) If $w = u + iv$ represent the complex potential function for an electric field and $u = 3x^2y - y^3$, determine the function v.</p> <p>4) State the Residue theorem and evaluate $\oint_C \frac{2z+1}{(2z-1)^2} dz$, where C is the circle $z = 1$.</p>
Q.28	<p>Attempt the following</p> <p>3) Prove that $f(z) = e^{2z}$ is analytic every in the plane and find its derivative.</p> <p>4) Expand $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = -2$.</p>
Q.29	<p>Attempt the following</p> <p>3) Evaluate $\int_0^{3+i} z^2 dz$ along (i) the line $y = \frac{x}{3}$ (ii) the parabola $x = 3y^2$</p> <p>4) Find the image of the upper half plane under the transformation $w = \frac{z}{i-z}$.</p>

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Q.30	<p>Attempt the following</p> <p>3) Find the Bi-linear transformation which maps the points $z = 1, i, -i$ into the Points $w = 0, 1, \infty$.</p> <p>4) Determine the analytic function whose real part is $y + e^x \cos y$.</p>
Q.31	<p>Attempt the following</p> <p>3) Evaluate $\int_C \frac{z^2 + 1}{z(2z + 1)} dz$ where C is $z = 1$.</p> <p>4) Under the transformation $w = \frac{1}{z}$ find the image of $x^2 - y^2 = 1$.</p>
Q.32	<p>Attempt the following</p> <p>3) Find the analytic function whose imaginary part is $e^x \sin y$</p> <p>4) Under the transformation $w = \frac{1}{z}$ find the image of $z - 2i = 2$.</p>
Q.33	<p>Attempt the following</p> <p>3) Evaluate $\int_C \frac{z^2 + 1}{z(2z + 1)} dz$ where C is $z = 1$.</p> <p>4) Under the transformation, $w = \frac{1}{z}$ find the image of $x^2 - y^2 = 1$.</p>
Q.34	<p>Evaluate</p> <p>(i) $\oint_C \frac{e^{2z}}{(z+1)^4} dz$; where $c: z = 4$</p> <p>(ii) $\oint_C \frac{e^z}{(z+1)^4(z-2)} dz$; where $c: z-1 = 3$</p>
Q.35	<p>Attempt the following.</p> <p>If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ find A^3 and A^{-1} using Cayley Hamilton Theorem.</p>

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	<p>Show that the matrix $\frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary.</p>
Q.36	<p>Attempt the following.</p> <p>Using Cayley-Hamilton theorem, find the inverse of $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$.</p> <p>If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I = 0$, Where I is a unit matrix of second order.</p>
Q.37	<p>Attempt the following.</p> <p>Define Hermitian matrix. If $A = \begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$ show that AA^* is a Hermitian matrix.</p> <p>Using Gauss -Jordan Method , find the inverse of $\begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.</p>
Q.38	<p>Find the eigenvalues & eigenvectors of the following matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.</p>
Q.39	<p>Find the eigenvalues & eigenvectors of the following matrix</p> $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$

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Q.40	<p>Attempt the following.</p> <p>Find A^{-1} by Gauss Jordan Method, where $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$.</p> <p>Find characteristic equation ,eigen value and eigen vectors of matrix A ifA= $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.</p>
Q.41	<p>If $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ find A^{-1} using Cayley Hamilton Theorem.</p>
Q.42	<p>If $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ find A^{-1} using Cayley Hamilton Theorem.</p>
Q.43	<p>Verify cayley-Hamilton theorem for the matrix A, where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$</p>
Q.44	<p>Verify cayley-Hamilton theorem for the matrix A, where $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$</p>
Q.45	<p>Verify cayley-Hamilton theorem for the matrix A, where $A = \begin{bmatrix} 3 & 2 & 4 \\ 4 & 3 & 2 \\ 2 & 4 & 3 \end{bmatrix}$</p>

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Q.46	<p>Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$</p>
Q.47	<p>Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$</p>
Q.48	<p>Attempt the following.</p> <p>Prove that the matrix $A = \begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{bmatrix}$ is unitary and find A^{-1}.</p> <p>Show that $A = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ is a Hermitian matrix.</p>
Q.49	<p>Attempt the following.</p> <p>Show that the matrix $A = \begin{bmatrix} \alpha+i\gamma & -\beta+i\delta \\ \beta+i\delta & \alpha-i\gamma \end{bmatrix}$ is unitary matrix, if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$.</p> <p>Show that every square matrix can be uniquely expressed as $P+iQ$, where P and Q are Hermitian matrices.</p>
Q.50	<p>Evaluate the line integral $\int_C [(y+3z)dx + (2z+x)dy + (3x+2y)dz]$ where C is the square formed by the lines $y = \pm 1$ and $x = \pm 1$.</p>

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Q.51	Evaluate the line integral $\int_C [(x^2 + xy)dx + (x^2 + y^2)dy]$ where C is the square formed by the lines $y = \pm 1$ and $x = \pm 1$.
Q.52	Verify Green's theorem for $\oint_C (x + y)dx + 2xy dy$, Where C is the boundary of the region bounded by $x = 0, y = 0, x = a, y = b$.
Q.53	Using Green's Theorem, evaluate $\int_C (x^2 y dx + x^2 dy)$, where C is the boundary described counter clockwise of the triangle with vertices (0, 0) (1, 0) & (1, 1).
Q.54	Verify Green's theorem for $\oint_C (3x - 8y^2)dx + (4y - 6xy)dy$, Where C is the boundary of the region bounded by $x = 0, y = 0$ and $x + y = 1$.
Q.55	Verify Divergence's theorem for $\vec{F} = 4xyz \hat{i} - y^2 \hat{j} + yz \hat{k}$ over the cube bounded by the planes $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$.
Q.56	Verify Gauss Divergence theorem for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallopied $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$.
Q.57	Evaluate $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ using Stoke's theorem when $\vec{F} = (2x - y, -yz^2, -y^2z)$. Where S is the upper hemisphere of the sphere $x^2 + y^2 + z^2 = a^2$ and C is its boundary.
Q.58	Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ by Stoke's theorem, where $\vec{F} = y^2 \hat{i} + x^2 \hat{j} - (x + z)\hat{k}$ and c is the boundary of the triangle with vertices at (0,0,0),(1,0,0)&(1,1,0).
Q.59	Evaluate $\oint_C (xy dx + xy^2 dy)$ by Stoke's theorem taking C to be a square in the xy-plane with vertices (1,0),(-1,0),(0,1) and (0,-1)

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Q.60	Evaluate the line integral $\int_C [(y + 3z)dx + (2z + x)dy + (3x + 2y)dz]$ where C is the square formed by the lines $y = \pm 1$ and $x = \pm 1$.
Q.61	Evaluate the line integral $\int_C [(x^2 + xy)dx + (x^2 + y^2)dy]$ where C is the square formed by the lines $y = \pm 1$ and $x = \pm 1$.
Q.62	Verify Green's theorem for $\oint_C (x + y)dx + 2xy dy$, Where C is the boundary of the region bounded by $x = 0, y = 0, x = a, y = b$.
Q.63	Using Green's Theorem, evaluate $\int_C (x^2 y dx + x^2 dy)$, where C is the boundary described counter clockwise of the triangle with vertices (0, 0) (1, 0) & (1, 1).
Q.64	Verify Green's theorem for $\oint_C (3x - 8y^2)dx + (4y - 6xy)dy$, Where C is the boundary of the region bounded by $x = 0, y = 0$ and $x + y = 1$.
Q.65	Verify Divergence's theorem for $\vec{F} = 4xyz \hat{i} - y^2 \hat{j} + yz \hat{k}$ over the cube bounded by the planes $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$.
Q.66	Verify Gauss Divergence theorem for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallopied $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$.
Q.67	Evaluate $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ using Stoke's theorem when $\vec{F} = (2x - y, -yz^2, -y^2z)$. Where S is the upper hemisphere of the sphere $x^2 + y^2 + z^2 = a^2$ and C is its boundary.
Q.68	Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ by Stoke's theorem, where $\vec{F} = y^2 \hat{i} + x^2 \hat{j} - (x + z)\hat{k}$ and c is the boundary of the triangle with vertices at (0,0,0),(1,0,0)&(1,1,0).
Q.69	Evaluate $\oint_C (xy dx + xy^2 dy)$ by Stoke's theorem taking C to be a square in the xy-

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	<p>plane with vertices (1,0),(-1,0),(0,1) and (0,-1)</p>
Q.69	<p>(a) Find the directional derivative of $\phi = 2x^2 + 3y^2 + z^2$ at point (2,1,3) in the direction of the vector (1,-2,0).</p> <p>(b) Find the directional derivative of $\phi = xy^2 + yz^3$ at point (1,-1,1) in the direction of the vector (1,2,2).</p>
Q.69	<p>(a) Find the directional derivative of $\phi = xy^2 + yz^3$ at point (1,-1,1) along the upward normal to the Surface $x^2 + y^2 + z^2 = 9$ at (1,2,2).</p> <p>(b) Find the directional derivative of $\frac{1}{r}$ in the direction of \vec{r}, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.</p>
Q.70	<p>(a) An electron moves such that its velocity is always perpendicular to its radius vector .Show that its path is circle.</p> <p>(b) Find the velocity and acceleration of the particle which moves along the curve $x = 2 \sin 3t, y = 2 \cos 3t, z = 8t, t > 0$. Also find the magnitude of the velocity and acceleration</p>
Q.71	<p>(a) Show that $\text{div}(\phi\vec{A}) = \phi(\text{div}\vec{A}) + (\text{grad}\phi)\cdot\vec{A}$.</p> <p>(b) Prove that $\text{div}(\text{grad } r^n) = \nabla^2(r^n) = n(n+1)r^{n-2}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.</p>
Q.72	<p>(a) Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$, where $\vec{r} = x\bar{i} + y\bar{j} + z\bar{k}$.</p> <p>(b) For a constant vector \vec{a}, show that $\nabla \times \left(\frac{\vec{a} \times \vec{r}}{r^3}\right) = -\frac{\vec{a}}{r^3} + \frac{3(\vec{a} \cdot \vec{r})}{r^5}\vec{r}$, where $\vec{r} = x\bar{i} + y\bar{j} + z\bar{k}$.</p>
Q.73	<p>(a) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$, where $\vec{r} = x\bar{i} + y\bar{j} + z\bar{k}$.</p>

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	(b) Show that the vector field $\vec{F} = \frac{\vec{r}}{r^3}$ is irrotational as well as solenoidal
Q.74	(a) A fluid motion is given by $\vec{v} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$. Is the fluid motion irrotational? (b) Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\mathbf{i} + (3xz + 2xy)\mathbf{j} + (3xy - 2xz + 2z)\mathbf{k}$ is both irrotational and solenoidal
Q.75	Show that $\vec{F} = (y \sin z - \sin x)\mathbf{i} + (x \sin z + 2yz)\mathbf{j} + (xy \cos z + y^2)\mathbf{k}$ is irrotational. Find scalar field ϕ such that $\vec{F} = \nabla \phi$
Q.76	Show that $\vec{F} = (6xy + z^3)\mathbf{i} + (3x^2 - z)\mathbf{j} + (3xz^2 - y)\mathbf{k}$ is irrotational. Find scalar field ϕ such that $\vec{F} = \nabla \phi$
Q.77	Attempt the following. 1) Express the function $f(x) = \begin{cases} 1; & x < 1 \\ 0; & x > 1 \end{cases}$ as a Fourier integral. Hence evaluate $\int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda.$ 2) Find the Fourier sine transform of $e^{- x }$. Hence show that $\int_0^{\infty} \frac{x \sin mx}{i + x^2} dx = \frac{\pi e^{-m}}{2}; m > 0.$
Q.78	Attempt the following. 1) Find the Fourier Transform of $f(x) = \begin{cases} 1; & x < 1 \\ 0; & x > 1 \end{cases}$. Hence evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$. 2) Find the Fourier integral represent for $f(x) = \begin{cases} 1 - x^2; & x \leq 1 \\ 0; & x > 1 \end{cases}$.
Q.79	Attempt the following. 1) Find the Fourier integral represent for $f(x) = \begin{cases} e^{ax}; & \text{for } x \leq 0, a > 0 \\ e^{-ax}; & \text{for } x \geq 0, a < 0 \end{cases}$. 2) Find the Fourier cosine transform of $f(x) = \begin{cases} 1; & 0 \leq x < 2 \\ 0; & x \geq 2 \end{cases}$

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Each question is of equal Marks (10 Marks)

Q.80	<p>Attempt the following.</p> <p>1) Using Fourier sine transform of e^{-ax} ($a > 0$), show that</p> $\int_0^{\infty} \frac{x \sin kx}{a^2 + x^2} dx = \frac{\pi e^{-ak}}{2} (k > 0) .$ <p>2) Using Fourier Integral show that $\int_0^{\infty} \frac{\omega \sin x\omega}{1 + \omega^2} d\omega = \frac{\pi e^{-x}}{2} (x > 0) .$</p>
Q.81	<p>Attempt the following.</p> <p>1) Find the Fourier sine and cosine transform of x^{n-1} ($n > 0$).</p> <p>2) Using Fourier integral representation, show that</p> $\int_0^{\infty} \frac{\sin \omega \cos \omega x}{\omega} d\omega = \frac{\pi}{2} (0 \leq x < 1) .$
Q.82	<p>Solve by Cardan's method $x^3 - 3x^2 + 12x + 16 = 0$.</p> <p>Solve the equation $x^4 - 12x^3 + 41x^2 - 18x - 72 = 0$ by Ferrari's method.</p>
Q.83	<p>Solve by Cardan's method $28x^3 - 9x^2 + 1 = 0$.</p> <p>Solve the equation $x^4 - 2x^3 - 5x^2 + 10x - 3 = 0$ by Ferrari's method.</p>
Q.84	<p>Solve by Cardan's method $x^3 - 27x + 54 = 0$.</p> <p>Solve the equation $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$ by Ferrari's method.</p>
Q.85	<p>Solve by Cardan's method $x^3 - 18x + 35 = 0$.</p> <p>Solve the equation $x^4 - 10x^2 - 20x - 16 = 0$ by Ferrari's method.</p>
Q.86	<p>Solve by Cardan's method $2x^3 - 5x^2 + x - 2 = 0$.</p> <p>Solve the equation $x^4 - 8x^3 - 12x^2 + 60x + 63 = 0$ by Ferrari's method.</p>